# SHRI RAMSWAROOP MEMORIAL UNIVERSITY

## End Semester Examination (2021-22)-Odd Semester

| M.Sc.(Mathematics)-I Year (I SEM) | |
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| Course Name:Algebra-I | Code: MMA1001 |
| Time: 02 Hours | Max Marks: 60 |

| **University Roll No.** | | | | | | | | | | | | | | | |
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| **(To be filled by the Student)** | | | | | | | | | | | | | | | |

**Note: Please read instructions carefully:**

a) *The question paper has 03 sections and it is compulsory to attempt all sections.*

b) *All questions of Section A are compulsory; questions in Section B and C contain choice.*

| **Section A: Very Short Answer Type Questions**<br>**Attempt all the questions.** | | **BL** | **CLO** | **Marks (10)** |
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| 1. | Define homomorphism with example. | BL1 | CLO1 | 02 |
| 2. | Define algebraic extension of field. What is the degree of $\sqrt{2}\sqrt{3}$ over $Q$. | BL1 | CLO4 | 02 |
| 3. | A group $G$ is nilpotent and iff $\gamma_{n+1}(G) = \{1\}$ for some integer $n \geq 0$. | BL2 | CLO2 | 02 |
| 4. | Show that the polynomial $f(x) = x^4 + x^3 + x^2 + x + 1$ is irreducible over $Q$ | BL2 | CLO3 | 02 |
| 5. | State Zassenhaus Lemma. | BL2 | CLO2 | 02 |
| **Section B: Short Answer Type Questions**<br>**Attempt any 03 out of 06 questions.** | | **BL** | **CLO** | **Marks (30)** |
| 1. | If $H$ and $K$ are normal subgroups of $G$, then prove that $HK = \{hk : h \in H, k \in K\}$ is a normal subgroup of $G$. | BL3 | CLO1 | 10 |
| 2. | State and prove Scherier's theorem. | BL1 | CLO2 | 10 |
| 3. | If $N$ is a normal subgroup of $G$ such that $N$ and $G/N$ are solvable then show that $G$ is also solvable. | BL2 | CLO2 | 10 |
| 4. | If $L$ is an algebraic extension of $K$ and if $K$ is an algebraic extension of $F$, then show that $L$ is an algebraic extension of $F$. | BL3 | CLO4 | 10 |
| 5. | Prove that a group of order 56 is not simple. | BL1 | CLO1 | 10 |
| 6. | Prove that for $r \in R, m \in M$ then,<br>$0.m = 0$. | BL1 | CLO5 | 10 |
| **Section C: Long Answer Type Questions:**<br>**Attempt any 01 out of 04 questions.** | | **BL** | **CLO** | **Marks (20)** |
| 1. | State and prove Sylow's third Theorem. | BL1 | CLO2 | 20 |
| 2. | Define central series. Show that a normal series $G = G_0 \geq G_1 \geq G_2 \geq - - - - \geq G_n = \{1\}$ is a central series if and only if $[G_i, G] \leq G_{i+1}$. | BL2 | CLO2 | 20 |
| 3. | Prove that an element $a \in K$ is algebraic over $F$ if and only if $F(a)$ is a finite extension of $F$. | BL2 | CLO3 | 20 |

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| 4. | Let $\bar{G}$ be the group of non-zero real numbers under multiplication and $G = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in R and ad - bc \neq 0\right\}$ be a group under matrix multiplication. Exhibit a homomorphism of $G$ on to $\bar{G}$. | BL3 | CLO2 | 20 |

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